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ABSTRACT

The purpose of the paper is to define for potential users of vocational education management information systems a quantitative analysis technique and its utilization to facilitate more effective planning of vocational education programs. Defining linear programming (LP) as a management technique used to solve complex resource allocation problems in business, government, and industry, this paper also discusses the potential of LP for vocational education planning, its relationship to a vocational educational management information system, and the strengths and weaknesses of linear programming for vocational education planning. A preliminary vocational education planning model illustrates the dynamic aspects and variations in the model and offers a representative model output. Finally, strategy is proposed for the development and implementation of the model. A seven page appendix offers a static preliminary State vocational education linear programming planning model. (Author/MW)

Research and Development Series No. 93.

# LINEAR PROGRAMMING FOR VOCATIONAL EDUCATION PLANNING

*Interim Report*



THE CENTER FOR VOCATIONAL AND TECHNICAL EDUCATION

THE OHIO STATE UNIVERSITY
1960 Kenny Rd., Columbus, Ohio 43210

## MISSION OF THE CENTER

The Center for Vocational and Technical Education is an independent unit on The Ohio State University campus. It serves a catalytic role in establishing consortia to focus on relevant problems in vocational and technical education. The Center is comprehensive in its commitment and responsibility, multidisciplinary in its approach, and interinstitutional in its program.

The Center's mission is to strengthen the capacity of state educational systems to provide effective occupational education programs consistent with individual needs and manpower requirements by:

- Conducting research and development to fill voids in existing knowledge and to develop methods for applying knowledge
- Programmatic focus on state leadership development, vocational teacher education, curriculum, and vocational choice and adjustment
- Stimulating and strengthening the capacity of other agencies and institutions to create durable solutions to significant problems
- Providing a national information storage, retrieval, and dissemination system for vocational and technical education

Research and Development
Series No. 93

# LINEAR PROGRAMMING FOR VOCATIONAL EDUCATION PLANNING

Interim Report

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December 1973

# FOREWORD

Resource allocation decisions required of state directors of vocational education continue to be difficult and challenging as they attempt to meet the needs of both individuals and society. This is in part due to the need for improved evaluative information. To assist in meeting this need, The Center for Vocational and Technical Education is in the process of developing a Management Information System for Vocational Education (MISVE). MISVE brings together for analyses a quantity of specific data needed to support management decisions through the identification of needs, opportunities, and problems. Broadly speaking, the information includes analyses of the labor market, demography, program costs, and impact on students. Some kind of mechanism is needed to synthesize and evaluate the large number of variables that become involved in such a comprehensive view. Linear programming, as a methodology, is discussed in this report.

We are indebted to the authors who collaborated in the preparation of this report: Robert C. Young, formerly a research and development specialist at The Center, now with the International Labor Organization, Geneva, Switzerland; Stanley Zionts, professor of management, European Institute for Advanced Studies in Management, Brussels, Belgium; and Albert B. Bishop, professor of industrial and systems engineering, The Ohio State University. The authors also acknowledge the helpful comments of Henry M. Levin, associate professor, School of Education and Department of Economics, Stanford University.

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Robert E. Taylor
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The Center for Vocational
and Technical Education

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# LINEAR PROGRAMMING FOR VOCATIONAL EDUCATION PLANNING

# INTRODUCTION

## Purpose of This Paper

The purpose of this paper is to define for potential users of vocational education management information systems a quantitative analysis technique and its utilization to facilitate more effective planning of vocational education programs. The nature of linear programming (LP) will be defined, as well as its potential for vocational education planning, its relation to a vocational education management information system, the strengths and weaknesses of LP for vocational education planning, variations on the LP theme that enhance its flexibility and utility to the user, and a proposed strategy for the development and implementation of the model.

## What is Linear Programming?

Linear programming (LP) is a management technique used for more than twenty years to solve complex resource allocation problems in business, government, and industry. It can be used to solve problems of minimizing cost or maximizing a measure of performance or effectiveness (usually called the objective function) subject to limitations on resource availability and other constraints. LP also may be used to analyze the consequences for the value of the objective function (e.g., the number of students achieving either prescribed skill levels, entry-level jobs, or minimum levels of job satisfaction) of alternative patterns of resource allocation or policy constraints within which the system might operate (e.g., no more than a 10 percent reduction in enrollments for any vocational education service area or specific program within a service area). Linear programming can consider alternative objective functions, analysis of policies to maximize alternate objective functions, and the consequences of changes in objective functions, levels of resource availability, and other policy constraints. These LP techniques assume, of course, that the relevant variables are known and that at least rough approximations are available concerning the relationships between variables.

## Who Has Used Linear Programming?

Among the non-educational uses of linear programming (LP) is that of determining the lowest cost blends satisfying certain constraints in numerous industries: food, oil, steel, and so on. The animal feed mix problem is an example of such a problem, the objective being to determine the lowest cost feed having minimum nutritional levels specified as well as having upper and lower limits on such factors as calories and weight. LP has also been used to allocate production of a large company to different plants, taking into consideration production facility limitations, cost of production, and transportation. The uses are too numerous to enumerate, but a good bibliography is available (Gass).

Linear programming has also been used for the analysis of educational resource allocations. Bowles, for example, constructed a linear programming formulation for northern Nigeria in which the private returns to education are maximized subject to limitations on teacher availability (allowing for importation and teacher training), flow constraints requiring that students complete one year of study before undertaking the next, and limitations on other resources. Bowles also includes constraints on social or public expenditures, initial student availability, and market demand for graduates. Using parametric programming (see section below: "Variations in the Model"), all solutions to the problem as the total present value of total social expenditure changes can be found.

Bruno proposed using linear programming to replace simplistic formulas for allocating state funds among school districts. His constraints specify that a minimum foundation level of support must be met, either from state funds or from local tax revenues. There are limitations on funds available, as well as maximum and minimum percentage levels of state and local funds to be used overall. He also proposes constraints for limiting the payment above the state support levels, among other constraints. The California Junior College state support program was studied, using the model, and it was found that it was possible to equalize the expenditure over all districts, thereby increasing the minimum expenditure level per student by about 2.5 percent without using additional resources.

McNamara proposed a linear programming model for the allocation of vocational education funds. He estimates the demand for and supply of graduates of non-vocational education (those trained formally in public or private institutions) in each occupational field. Additional graduates may be trained via vocational education; the residual market for graduates is assumed to be known for each program. The objective is to maximize the number of individuals trained, subject to the constraints of school capacity, funds, teachers, and so on. No mobility is assumed. The model is applied to data for the Philadelphia labor market area for the years 1969-1971. The principal weaknesses of this model are its assumptions—if the objective function is to be worthy of maximization—that training related placements reflect relative economic gains for students, that student interests will be sufficient to fill all training slots funded, and that the economic, psychological, or other benefits generated for the individual or society are identical, regardless of the program in which the student enrolls.[1]

### Linear Programming and Management Information Systems

The quality of the output from linear programming models can be no better, of course, than the quality of the inputs into the computer, including the quality of the model itself, the reasonableness

[1]For additional uses of linear programming in education, see ERIC documents ED 61283, ED 59524, ED 52526, ED 52527 (similar to 52526), ED 51563, ED 46250, ED 28525, ED 26736, ED 30430, ED 14912, ED 14914, ED 14915, ED 18114, ED 18957, ED 19884, and ED 20677. After stating, "To date, almost no use has been made of linear programming for the solution of problems in education," Van Dusseldorp, Richardson, and Foley, indicate they expect "its use will rapidly increase as educators become more familiar with this tool."

of the resource constraint assumptions, and the nature of the (alternative) objective function(s). These inputs may be derived entirely from an empirical data base, when data are unavailable. Best guesses (based on surveys, Delphi, gaming, interviews, etc.) may serve as reasonable proxies until better empirical data are available. As a LP model evolves, or picks up sophistication, the data requirements will gradually increase. Initially, a model might take the form of the McNamara model. To this might be added information in a priority order such as the following: first, data on vocational student course preferences; second, aptitude data; third, follow-up information on vocational student earnings and job satisfaction; fourth, data from general, vocational, and college preparatory student follow-ups, etc.[2] The use of the model while it is evolving from germination to full fruition will be discussed below. The important point to be made here is that the LP model will help structure the organization of masses of data into constraints and an objective function. Without this structure, the approach to decisions may be haphazard and the sheer volume of data might overwhelm the decision-maker. Using LP, the decision-maker will be assisted by his interaction with the computer and his capability will be enhanced to identify needed data and generally make better decisions. Any fears held by the decision-maker of being replaced by the computer are nonsense.

## Outline of the Paper

Subsequent sections will discuss several aspects of linear programming for vocational education planning. The following questions will be examined: What is the meaning of "linear" programming? What are the roles of constraints in the model? What will the overall model look like? What are the strengths of the model? What are the weaknesses of the model? And, finally, what are reasonable steps to be followed in building and implementing the model?

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[2]Oklahoma's State Department of Vocational and Technical Education is currently developing a LP model using the U.S. Department of Labor's General Aptitude Test Battery scores to reflect aptitudes and so provide an aptitude constraint for program funding.

# ON LINEAR PROGRAMMING

## The Meaning of "Linear" Programming

Linear programming is so called because the relationships between the inputs into a system and the outputs from that system may be depicted graphically through straight line relationships. This means that linear programming can be used provided that the objective to be maximized and the constraints upon maximization can be expressed as linear equalities and inequalities. In other words, every unit of a particular product must require the same amount of each resource for production and contribute the same amount to the function being maximized or minimized. An example of this would be where the addition of $30,000 might be necessary to support an additional thirty students regardless of whether the state had ten thousand or twenty thousand students currently on its roster. If cost per capita increased disproportionately with the increase of enrollment, so that linear assumptions would be operationally unsatisfactory, linear programming might be inappropriate.[3]

## The Objective Function

The objective function is that output from the system that is to be maximized (or minimized, if the objective function is a cost function). In industry there is general agreement that long-run profits (in some sense) might be the objective function to be maximized by the firm. In vocational education, there would be substantially more debate about the objective: Is it job satisfaction for the students? Income? Training-related placements? Dropout reduction? Social welfare? Other? Or some index number reflecting a weighted average of several kinds of impact scores?

An important related question is the extent of the resource allocations to maximize alternative objective functions. This is one of the benefits of linear programming: The model can tell us, given an appropriate data base or best reasonable guesstimate[4] for the impact of particular programs, whether maximizing income will also tend to maximize job satisfaction, training-related placements, etc., and the number of students who satisfactorily complete high school. Similarly, it would tell us the amount of income that would be sacrificed by not maximizing income if we maximize job satisfaction.

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[3]In certain circumstances, such as when logarithmic or piecewise linear functions may be used, non-linear relationships may be modified for linear programming. See, for example, Hadley or Zionts.

[4]One might use the most optimistic, expected, and most pessimistic guesses to provide estimates for the probable limits on the impact of programs. The techniques for LP under uncertainty have already been developed, and more precise statistical methods may be used in certain cases.

### Multiple Objectives

In addition to using LP for telling us the allocations for maximizing one objective function rather than another, LP may also be used to indicate maximization allocations for one objective function, given that we want to achieve minimum satisfactory levels of output in other dimensions. For example, a well-developed LP might suggest the allocation that maximizes entry wages for graduates subject to the constraints, say, of a minimum training-related placement rate, a maximum cost per graduate, and/or some minimum level of job satisfaction. This is one way of dealing with the problem of multiple objectives[5] and does, of course, require trade-offs that cannot be determined solely through the use of the computer itself but must be made by the decision-makers. Neither the computer nor linear programming can avoid the bargaining process, but the LP model can display for the decision-makers the consequences of their "bargains."

### Problem Constraints

Constraints, in linear programming, are restrictions imposed on the system, or its linear model, as it attempts to maximize the output reflected in its objective function. Constraints that might be

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[5]Another means for dealing with the problem of multiple objectives is to develop an index number for the impact of the program on several objectives. The curricula might be ranked according to their impact on each of the criteria, the ranks summed (see Figure 1 below) and the LP might then maximize the sum of the products of the summary impact index times the number of students terminating from the respective program.

Fig. 1 CURRICULAR IMPACT RANKS AND INDICES AND PRIORITY RANKS (HYPOTHETICAL)

| Curricula / Ranking* Criteria | Wages | Job Satisfaction | Related Placement | Summary Impact Index | Priority Rank |
|---|---|---|---|---|---|
| Nurse | 3 | 4 | 4 | 11 | 1 (Highest) |
| Filing Clerk | 1 | 1 | 2 | 4 | 4 (Lowest) |
| Child Care | 2 | 3 | 1 | 6 | 3 |
| Programmer | 4 | 2 | 3 | 9 | 2 |

*Cell scores reflect the ranks of the curricula's impacts upon the columns' evaluation criteria.

For further discussion, see Young, Clive, and Miles.

imposed on a linear program for vocational education planning include dollar resources, teachers, classroom space, interested students, minimum acceptable average entry wages for graduates, maximum expenditure levels per graduate or terminee, maximum rates of program reduction, esrimated net occupational openings related to the training curricula, student aptitudes, and the manner in which certain funds may be expended. The decision-makers and planning analysts in the state must decide which constraints are real and fixed and which may be subject to influence.[6] These constraints may then be specified in the overall model as limitations within which the planners and administrators must operate to maximize the objective function.[7]

## Weaknesses of Linear Programming

One of the big problems in using linear programming for vocational education planning is the necessity of developing a data base and analytical capacity to support an LP model that will be more than superficial and avoid dysfunctional allocations. Development of such data bases is expensive and requires first-rate management to design the data collection, organization, and analysis so that it is supportive rather than dominant in planning. While such data bases may be costly, if management is committed to a sophisticated management information system, the additional cost (beyond the cost of the data base) of linear programming analysis becomes relatively modest.

Another problem, referred to elsewhere in this paper, is that relationships between the inputs and the outputs of the system must be such that they may be modeled using linear functions. There are instances where nonlinear relationships may be reasonably approximated for LP purposes through the use of piecewise linear or logarithmic functions.

Finally, comprehension of linear programming output is not necessarily instinctive, and it will be necessary for the analysts to train management to understand the data displays and/or the significance of selected data from the programs. This is a concern not to be overlooked and is facilitated if the model is developed with the close cooperation of the planner or model-builder and the decision-maker.

## Strengths of Linear Programming

Most of these strengths are mentioned elsewhere but will be reiterated here. The basic argument in its support is, of course, that the model forces the decision-makers and analysts to think about the

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[6]For example, if legislators are shown that vocational education enhances the state fiscal position, more vocational education resources may be made available.

[7]Whereas in solving a system of equations for a unique solution, in general, the number of unknowns must be equal to the number of equations, in LP the number of unknowns (alternative processes, "activities", or "decision variables" in LP terminology, to which resources will be allocated) must be greater than the number of constraints expressed by the equations.

relationships between the inputs and the outputs. As a result of careful analyses of these relationships, and with the assistance of the computer in synthesizing torturous masses of data, linear programming will facilitate better decisions by the decision-maker and thus more cost-effective programs. Basically, the way this cost-effectiveness and improved decision-making are facilitated is by facilitating the discovery, through iterations with the model, of the consequences of alternative planning decisions. Through these iterations, the decision-maker is able to move closer to the maximization of the objectives he has specified within the constraints he has identified. This efficiency may come from unsuspected consequences of trade-offs or from the analysis of the benefits of yet untried alternatives.

A secondary benefit from LP is that, because the model must be developed through iterations with the decision-makers, the model will also facilitate the determination of data that are necessary and unnecessary for planning purposes. Consequently, some costs may be saved due to the elimination of unnecessary data, and critical data oversights may also be identified. Finally, as the modeling and planning staff gain the confidence of the key decision-makers, the model-builders will find it easier to use the vast experience of the decision-makers and incorporate it into the model. Both the objective function and the constraints may then be carefully structured into a vocational education LP model.

# A PRELIMINARY VOCATIONAL EDUCATION PLANNING MODEL

Linear programming models evolve with the increasing sophistication of their related analyses and data bases and the enhanced specificity of the constraints in which the system operates. Although the ultimate objective of the project is to build as sophisticated a model as data and reasonable resources will support, at this stage only a simplified model will be presented. Alternative objective functions and constraints are presented for two basic reasons: first, different states may specify different goals and constraints, and, second, any state may want to examine the programmatic consequences of alternative objective functions or alternative levels or compositions of the constraints.

Initially the preliminary model's gross statewide and service area data inputs and outputs will be specified, with increasing sub-state and curricular code detail being added to the model over time. Time will also enable the collection and analysis of additional kinds of data (e.g., value added), which will incrementally enable the model to provide better outlines for the detailed guidance of the vocational system. The concepts that will be examined in both the preliminary and the sophisticated versions of the model are compared below, while the appendix describes the preliminary model in more elaborate algebraic form.

A few comments are in order regarding the format (Figure 2) for indicating the alternative objective functions, constraints, and coefficients for the activity levels in the object functions and the constraints. First, the initial objective functions are suggested for the reasons stated in the first paragraph of this section. Similarly for constraints, several are suggested and more, eventually, will be mandatory for the user of the model. Those objectives and constraints which are most critical may be tested, compared, rejected, or modified, within the limits of the back-up data system. The preliminary version of the model, however, will include objective functions that are widely agreed to be important. More extensive objectives and constraints will eventually be available.

## Dynamic Aspects of the Model

Linear programming can deal with the dynamic problem of allocating resources over time. Several points should be mentioned here. First, some means must be adopted for dealing with the problem of the relative value of benefits received in the near future as opposed to those likely in the more distant future. Discounting, with either an agreed upon or a range of discount rates, is the standard approach to this problem.

A second dynamic aspect is that longitudinal data become very important because of the entire set of private and public benefits due to vocational education. In the early phases of the project,

(text continued on page 14)

Fig. 2 MODEL CONCEPTS

| Preliminary Model | | Sophisticated Model | |
|---|---|---|---|
| | ALTERNATIVE OBJECTIVE FUNCTIONS | | |
| IA | Maximize the aggregate entry earnings of terminees for the year following the current planning year. | I | Maximize the net value added to discounted terminee earnings, controlling for earnings of similar students from the general and/or college preparatory curriculum, by USOE code or course. |
| IB | Maximize the sum of the products, over all service areas, of {[(median earnings for terminees in the labor force) (number of terminees in the labor force or in higher education)] ÷ [enrollment in the service area]} {number of planned admissions}. | | |
| IIA | Maximize the number of students placed on training-related jobs. | IIA&B | Same, with greater detail within service areas. |
| IIB | Maximize the number of students placed on jobs, regardless of whether related or not. | | |
| IIIA | Maximize the number of students obtaining related jobs or continuing with related higher education. | IIIA&B | Same, with greater detail within service areas. |
| IIIB | Maximize the number of students obtaining jobs or continuing with higher education, regardless of whether related or not. | | |
| IV | Maximize the sum of job satisfaction scores. | IV | Same, with greater detail within service areas. |

(Continued)

Fig. 2 MODEL CONCEPTS (Continued)

| Preliminary Model | Sophisticated Model |
| --- | --- |
| **OBJECTIVE FUNCTIONS (Continued)** | |
| V Maximize the sum of the products, by service area, of the impact index times enrollments (this maximizes an objective function composed of several weighted outputs). | V Same, with greater detail within service areas and more sophisticated impact indexes. |
| — — — — — — | VI Maximize the net value added to the fiscal position of governments. |
| — — — — — — | VII Maximize the net social benefits above social costs for the vocational education system. |
| **CONSTRAINTS** | |
| Training-related forecasted openings, net of supply from other sources as provided by the state employment security agency. | Improved net openings data, which may include some information on on-the-job training and occupational mobility patterns. |
| Numbers of students who might reasonably be expected to enroll in specific vocational education programs during the relevant years. | Same, based on better empirical data. |
| Minimum proportion of funds that must be spent on disadvantaged students (15 percent for federal funds). | Same |
| Minimum proportion of funds to be spent on those who have completed or left high school (15 percent of federal funds). | Same |

(Continued)

| Preliminary Model | Sophisticated Model |
|---|---|
| **CONSTRAINTS (Continued)** | |
| Minimum proportion of funds to be spent on the handicapped (10 percent of federal funds). | Same |
| Total vocational education dollars available. | Dollar resources available, classified by type of expenditure permissible (e.g., capital as opposed to operating budgets). |
| - - - - - - - - - - - - - - - - - | Minimum satisfactory levels of job satisfaction, say, those levels achieved by the typical general education graduates. |
| - - - - - - - - - - - - - - - - - | Number of students who have completed one level (say, the junior year) and are available to enter the subsequent level the following year. |
| - - - - - - - - - - - - - - - - - | The dropout rate of the students. |
| - - - - - - - - - - - - - - - - - | The rate of entry or reentry of new or former students into the various levels of the system. |
| - - - - - - - - - - - - - - - - - | Aptitude scores of students, which may be compared to minimal levels necessary for adequate job performance. |
| - - - - - - - - - - - - - - - - - | Teacher supply, for specific courses, where this is a significant constraint. |

(Continued)

Fig. 2 MODEL CONCEPTS (Continued)

| Preliminary Model | Sophisticated Model |
|---|---|
| ACTIVITY COEFFICIENTS | |
| Entry wage earned by terminee of a service area. | The net increase in earning power of graduates due to* specific vocational education USOE code programs, number of, and kind of courses taken. |
| Cost per FTE in each vocational education service area. | Cost per enrollee in each specific vocational education course. |
| Job satisfaction of students by program areas. | Job satisfaction of students due to particular courses: |
| Student curricula to jobs flow data, by service area. | Same, by program area. (USOE code). |
| ------------------- | The distribution of aptitude scores by students who take jobs in particular occupations. |
| ------------------- | Classroom and other capital requirements such as equipment, including their costs, for specific courses. |
| ------------------- | Impact upon the fiscal position of the state due to the course, program, and service area. |
| ------------------- | Net contribution to the economic welfare of the community due to the course, program, and service areas. |
| ------------------- | Effect upon training related placement rate of the program due to investment in specific courses. |
| ------------------- | Student flow data, on the sources of students for particular classes. |

*"Due to" is used here in the sense of "statistically determined by" rather than reflecting a cause and effect relationship.

short-term follow-up data, perhaps supplemented with analyses from comparable longitudinal studies (such as that of the Ohio State University Center for Human Resource Research), can be utilized as proxies for long-term benefits. But this short-term data should later be replaced as long-term follow-ups are implemented.

A third point, regarding the model's time dimension, is that it is important to deal with vocational education's capital and operating budgets simultaneously so that allocations may be made immediately for those programs that will require capital facilities in the future. This is the obvious trade-off between more operating programs now and more operating programs in the future. The model would be able to tell us, then, the optimal split between capital and operating funds and the cost of not so allocating resources between those budgets.

Finally, it should be mentioned that one of the beauties of such a model is that it forces the planner and decision-maker to recognize some dynamic and cost relationships that might otherwise be overlooked. For example, state divisions of vocational education often have information on annual cost per full-time equivalent by service area. To the extent that those cost figures and benefits from the respective programs are relatively similar for vocational education graduates across service areas, the model will tend to support those relatively short-term (say, one-, two-, or three-semester programs such as distributive education) rather than relatively long programs (such as a four-year agricultural education program). Of course, if the benefits of the three- or four-year program are sufficiently great to warrant funding over the longer period of time, then resources would be so allocated. In other words, the LP model would tend to underscore the importance of the length of time over which funds should be provided to a specific cohort of students. Unless benefits were sufficiently great to compensate for higher costs, shorter programs would be funded.

### Variations in the Model

As stated above, the basic output from a linear programming analysis is a set of resource allocations that should maximize the achievement of the goal(s) of the system. In addition to the optimum allocation among programs, LP is also able to point out to the user the consequences for program effectiveness of deliberately providing non-optimal levels of resources to programs.[8]

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[8]There are several kinds of such related analyses. One, the analysis of "changing limits," or the effect on output of a one unit change in the constraints on the right-hand side of the model's equations (see Appendix), referred to as the marginal value, also yields information on the range over which the marginal value applies. A second, "profit range" analysis, yields the ranges over which changing the objective function coefficients will not cause changes in the optimum activity levels. A third, "reduced cost" or reduced profit analysis, reveals the consequences for output of resources being provided to one unit of an activity not funded in the optimum set of programs. A fourth such analysis, of "trade-offs," examines the effect upon all the basic variables of a one unit change in an activity or a constraint. A fifth, "technological sensitivity" analysis, examines the sensitivity of the system's output to a small change in a coefficient of a constraint, A sixth such analysis, "parametric

Parametric programming is among the more useful of these analyses and enables the simultaneous examination of non-marginal changes in several constraints, the objective function, or any of the activities proposed for funding. In other words, linear programming and its variations can tell us not only what would be the optimal allocation but also, through its various subroutines, what would be the cost of small or large modifications in the optimal allocation of resources.

## Representative Model Output

While linear programming analyses are capable of providing a great deal of information, as discussed in the previous section, its analysis must be focused for the decision-maker. Thus, tables with some of the key output from LP analysis are presented in Figures 3 and 4. In Figure 3, column one represents the fact that the program may be run against various objectives, several of which are presented here. Column two represents the potential value of the objective function if one objective is maximized, or the impact on that secondary objective if another objective is maximized. Column three presents the loss in value for one objective if another is maximized. The fourth column, or set of columns, reflects the composition of enrollments (hypothetical) necessary to maximize that row's objective. Figure 3, then, emphasizes the fact that to maximize one objective, a price is paid (a trade-off) in terms of reducing the impact upon the other objectives. Figure 4 simply illustrates that as one changes the objective function to be maximized, a price is paid in terms of a reduction in the value of other outputs. The inability to simultaneously maximize all objective functions means that to maximize, say, placements imposes a cost (or trade-off) of $250,000 in wages, a reduction in average job satisfaction from 5.1 to 3.0, and forty related placements, compared to the possibilities if those other objective functions were maximized.

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programming," examines the responsiveness of output to non-marginal changes in more than one of the variables, changes in either the constraints, objective function, or activities included which may result in the addition or deletion of certain activities from the production process.

Fig. 3 LINEAR PROGRAMMING OUTPUT (Hypothetical)

| (1) IF MAXIMIZING: | (2) AGGREGATE ENTRY WAGES | (3) WAGES LOST DUE TO NOT MAXIMIZING WAGES | (4) PLAN ENROLLMENTS | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | AGR | B&O | DE | H | HE | T | T&I | Other |
| ENTRY WAGES | 1,000,000 | 0 | 30 | 70 | 30 | 50 | 20 | 80 | 110 | 10 |
| JOB SATISFACTION | 900,000 | 100,000 | 30 | 60 | 25 | 60 | 25 | 80 | 100 | 20 |
| PLACEMENTS | 825,000 | 175,000 | 10 | 95 | 50 | 80 | 60 | 50 | 75 | 15 |
| TRAINING RELATED PLACEMENTS | 750,000 | 250,000 | 5 | 85 | 40 | 70 | 10 | 75 | 105 | 10 |
| IF ENROLLMENT CHANGES IN PERCENT ARE AS IN PREVIOUS YEAR: | 750,000 | 250,000 | 100 | 90 | 25 | 15 | 60 | 50 | 50 | 10 |

Fig. 4 OBJECTIVE FUNCTION TRADE-OFFS (Hypothetical)

VALUE OF OTHER OUTPUTS:

| IF MAXIMIZING: | ENTRY WAGES | AVERAGE JOB SATISFACTION | PLACEMENTS | RELATED PLACEMENTS |
|---|---|---|---|---|
| ENTRY WAGES | $1,000,000 | 4.5 | 200 | 125 |
| JOB SATISFACTION | 900,000 | 5.1 | 200 | 140 |
| PLACEMENTS | 750,000 | 3.0 | 250 | 110 |
| TRAINING RELATED PLACEMENTS | 750,000 | 3.5 | 225 | 150 |

# BUILDING AND IMPLEMENTING THE MODEL

## Next Steps

Following general agreement by the program planners and administrators concerning the outline of a preliminary LP model such as the one proposed herein, several subsequent steps appear reasonable. A few comments will be made about each.

1. Elaborate upon, formulate, and solve a simplified problem of the type proposed here. Approximately two people will be needed for one or two months: a person knowledgeable in state vocational education operations and an operations research specialist. In addition, a computer time-sharing budget to cover approximately sixty minutes of computational time will be needed (not including the cost of the telephone or terminal). These estimates assume existence of the data, some manipulation of it into LP formats, and some debugging of programs.

2. Present the ideas in a one-day or half-day seminar to a group of about four or five state vocational education decision-makers—all from the same state, but not all from the same agency. Alter it and incorporate their suggested changes to the simple model <u>on the spot</u>, using conflicting objectives. Those involved in Step 1 as well as the decision-makers will be involved. About 15 minutes of time-shared computer service will be required.

3. Based on the result of Step 2, develop a larger scale version of the model (e.g., disaggregating by districts, USOE curricula, capital-operating budgets, and adding additional constraints). Develop a data-information system to support it. During this phase, maintain a close liaison with the state vocational education decision-makers. This step will require about six months' to one year's development on the part of two or three individuals (e.g., including an operations research specialist, and a vocational education planner) plus about 20 hours of computer time. At the end of this phase, a preliminary report will be prepared and the product will be presented to the state education agency. If the data files and formats are consistent, the model, because of its generality, could be developed for several states simultaneously with only moderate additional costs.

4. Work together with the state education agency to alter as necessary and implement the model. Prepare a final report for distribution to other state agencies. This step will require about two or three people for a six-month period, plus about 20 hours in computer time.

5. The eventual operational costs of a moderately sophisticated system (a 500-equation model), above the cost of a management information system (assuming this system to be in accessible form on tape), are estimated to be 5 hours in computer time and one man-month of the MIS systems analyst's time, in addition to the teletype terminal used for the rest of the MIS operations. These are conservative estimates based on approximately fifty runs of the data annually.

### Release of Model Output as It Evolves

The model will pick up sophistication in proportion to the quality of the improvement of its data base, the analyses of that data, and the manipulation of that data into LP formats. Some argue that some data is better than none, and therefore even preliminary data should be used if it is all that is available. However, because of the possibly dysfunctional effects that might follow from the use in planning of the output from the preliminary model, it is strongly recommended that crude LP outputs not be disseminated beyond the developmental staff, with the exception of dissemination for critique or discussion rather than for implementation purposes. Thus, results of early runs would be used primarily for feedback purposes, namely to guide evolution of the model from preliminary to more advanced forms and to indicate changes needed in the data base and information system. The appendix that follows is presented as a more algebraically specific version of the general preliminary model discussed above.

# APPENDIX

## A STATIC PRELIMINARY STATE VOCATIONAL EDUCATION LINEAR PROGRAMMING PLANNING MODEL

### I. ALTERNATIVE OBJECTIVE FUNCTIONS

In all cases, Z is the function to be maximized, subject to the constraints listed following the alternative definitions of Z. In all cases,

$$Z = \sum_{j=i}^{8} v_j x_j ,$$

where:
j = the subscript which ranges from one to eight and reflects the particular service area for which that variable is representative. The service areas and their representative values of j (and USOE code numbers) are as follows:

1 = agriculture (1.0000)
2 = distributive education (4.0000)
3 = health occupations (7.0000)
4 = home economics (9.0000).
5 = office occupations (14.0000)
6 = technical education (16.0000)
7 = trade and industrial occupations (17.0000)
8 = other

$v_j$ = the coefficient, for the $j^{th}$ service area, reflecting the impact for the latest follow-up year per admission or enrollment into that service area upon the value being maximized in that particular objective function; $x_j$ = the number of planned admissions into service area j during the implementation year estimated to contribute to the maximization of that particular objective function, within, of course, the constraint of student interest.

Because one of the objectives of all vocational education programs is to provide a reasonable number of students with services of some minimal benefits, administrative constraints might be built into the system to not provide funds to any program costing more than some maximum per FTE (say, \$2,000 or \$2,500) or where median wages of graduates f ll below some minimal level (say, \$1.60 per hour). These would then be constraints on all objective functions, the former avoiding excessive concentration of resources among a few students and the latter eliminating programs whose terminees fail to achieve a bare minimum of economic success. These constraints would not be likely to be effective in the preliminary model becuase of the aggregation of all programs within service areas, unless entire service areas served their students so poorly.

As only the $v_j$'s will differ among the alternative objective functions below, the rest of the definition of Z being identical to that above, the alternative objective functions will thus be defined in terms of $v_j$.

A. Objective Function IA: Maximize the aggregate entry earnings of terminees for the year following the current planning year.

$v_j$ = the median (mean) income for service area j, the list of those from whom the median (mean) is derived consisting of all terminees, including dropouts and graduates, employed as well as those unemployed and not in the labor force, the latter two groups obviously receiving O's to reflect their earnings.

B. Objective Function IB: Maximize the sum of the $v_j x_j$'s, where $v_j$ is defined as

$$v_j = \frac{\text{Median (Mean) earnings of terminees in the labor force} \times \text{Number of terminees in the labor force or in higher education}}{\text{Enrollment in the service area during the period for which the above data were collected}}$$

Note: This is essentially different from "IA" in that value is given not only to explicit earnings but also to those who go on to higher education, the assumption being that vocational education has contributed something of value when its graduates are able to continue in higher education. That value is considered the same as the earnings of their labor force peers and could, as an additional alternative, be weighted more or less.

C. Objective Function IIA: Maximize the number of students placed on training-related jobs.

$v_j$ = the ratio of students placed in related jobs to admissions/enrollments.

D. Objective Function IIB: Maximize the number of students placed on jobs, regardless whether related or not.

$v_j$ = the ratio of the number of students placed on jobs to the number of admissions/enrollments.

E. Objective Function IIIA: Maximize the number of students obtaining related jobs or continuing with related higher education.

$v_j$ = the ratio of the number of students entering either related education or related jobs to enrollment/admissions for that year.

F. Objective Function IIIB: Maximize the number of students obtaining jobs or continuing with higher education, regardless of whether the jobs or education are related directly to the training or not.

$v_j$ = the ratio of the number of students obtaining jobs or continuing with higher education to the number of admissions/enrollments.

G. Objective Function IV: Maximize the sum of job satisfaction scores.

$v_j$ = the ratio of total job satisfaction scores to the total number of admissions/enrollments.

H. Objective Function V: Maximize the sum of the products, by service area, of the impact index times enrollments.

$v_j$ = the impact index for service area j.

## II. BASIC DECISION VARIABLES

$D$ = designates a person as being disadvantaged.
$\bar{D}$ = designates a person as not being disadvantaged.
$O$ = designates a person as being out of high school.
$\bar{O}$ = designates a person as being in high school.
$H$ = designates a person as being handicapped.
$\bar{H}$ = designates a person as not being handicapped.

$x_j^{\bar{D}\bar{O}\bar{H}}$ = number of non-disadvantaged, non-handicapped, in high school people to be admitted per year to programs in service area j: j = 1, ---, J.

$x_j^{\bar{D}\bar{O}H}$

$x_j^{\bar{D}O\bar{H}}$

$x_j^{\bar{D}OH}$

$x_j^{D\bar{O}\bar{H}}$

$x_j^{D\bar{O}H}$

$x_j^{DO\bar{H}}$

$x_j^{DOH}$

Same for other combinations of being or not being disadvantaged, out of high school, and handicapped. Total number of variables = 8J (not including slacks).

$x_j^{doh}$ = general notation to refer to any given combination of being or not being disadvantaged, out of high school, and handicapped (referred to hereafter as "personal category 'doh' ").

Note: $x_j$, as used in the preceding discussion of objective functions is related to the $x_j^{doh}$ as follows:

$$x_j = \sum_{d=D,\bar{D}} \; \sum_{o=O,\bar{O}} \; \sum_{h=H,\bar{H}} x_j^{doh}$$

## III. CONSTRAINTS

### A. Forecasted Training-Related Openings in Area j

$M_j$ = annual number of net openings forecast for area j.

$a_{jk}^{doh}$ = fraction of students in personal category "doh" enrolled in programs in area k who take jobs in area j.

Constraints:

$$\sum_{k=1}^{J} \; \sum_{d=D,\bar{D}} \; \sum_{o=O,\bar{O}} \; \sum_{h=H,\bar{H}} a_{jk}^{doh} \; x_k^{doh} \leq M_j; \; j = 1, \ldots, J$$

Remark: If there is no cross-fertilization of areas, i.e., if none of those trained in area j seek employment in any other area, these constraints become

$$\sum_{d=D,\bar{D}} \; \sum_{o=O,\bar{O}} \; \sum_{h=H,\bar{H}} a_j^{doh} \; x_j^{doh} \leq M_j; \; j = 1, \ldots, J$$

where, $a_j^{doh}$ = the portion of enrollees from personal category "doh" in programs in area j who seek related vocational employment.

B. Availability of Total Number of Students

s = a set of service areas. The possible sets include all possible combinations of the J service areas, e.g., (1), (2), ---, (J), (1,2), --- (1,J), (2,3), ---(2,J), ---(J-1,J), (1,2,3) ---, (1,2,3, ---J). The number of such sets for J service areas is $2^J$-1, not counting the set corresponding to no interest in any area.

$N_s$ = the number of potential students with an interest in (and attributes for acceptance in) programs in the set of service areas s.

Constraints: There are also $2^J$ -1 constraints of the form:

$$\sum_{d=D,\bar{D}} \sum_{o=O,\bar{O}} \sum_{h=H,\bar{H}} x_j^{doh} \leq \sum_{\text{all s containing } j} N_s \qquad j=1, \text{---}, J$$

$$\sum_{d} \sum_{o} \sum_{h} (x_j^{doh} + x_k^{doh}) \leq \sum_{\text{all s containing j, k, or both}} N_s \qquad j, k=1, \cdots, J \quad j \neq k$$

$$\sum_{d} \sum_{o} \sum_{h} (x_j^{doh} + x_k^{doh} + x_l^{doh}) \leq \sum_{\text{all s containing j, k, l or combinations thereof}} N_s \qquad j, k, l=1, \text{---}, J \quad \text{j, k, l all different}$$

$$\vdots$$

$$\sum_{j=1}^{J} \sum_{d} \sum_{o} \sum_{h} x_j^{doh} \leq \sum_{\text{all s}} N_s$$

C. Availability of Subsets of Students (Disadvantaged, Out of High School, Handicapped)

$N^{doh}$ = the number of potential students available in personal category doh.

$C_j^{doh}$ = Average program cost to support one student in personal category "doh" in a program in service area j; j = 1, ..., J.

Constraints:

1) Disadvantaged:

$$.15 \left( \sum_{j=1}^{J} \sum_{d=D,\bar{D}} \sum_{o=O,\bar{O}} \sum_{h=H,\bar{H}} C_j^{doh} \, x_j^{doh} \right) \leq \sum_{j=1}^{J} \sum_{o=O,\bar{O}} \sum_{h=H,\bar{H}} C_j^{Doh} \, x_j^{Doh}$$

2) Out of High School:

$$.10 \left( \sum_{j=1}^{J} \sum_{d=D,\bar{D}} \sum_{o=O,\bar{O}} \sum_{h=H,\bar{H}} C_j^{doh} \, x_j^{doh} \right) \leq \sum_{j=1}^{J} \sum_{d=D,\bar{D}} \sum_{h=H,\bar{H}} C_j^{dOh} \, x_j^{dOh}$$

3) Handicapped:

$$.15 \left( \sum_{j=1}^{J} \sum_{d=D,\bar{D}} \sum_{o=O,\bar{O}} \sum_{h=H,\bar{H}} C_j^{doh} \, x_j^{doh} \right) \leq \sum_{j=1}^{J} \sum_{d=D,\bar{D}} \sum_{o=O,\bar{O}} C_j^{doH} \, x_j^{doH}$$

D. <u>Budget</u> (Funds to support the programs)

$C_j^{doh}$ = average program cost to support one student in personal category "doh" in a program in service area j; j=1, —, J.

B = Annual budget available to support all vocational education programs.

Assumptions:

1. Once a student enters a program, the cost incurred is the same whether he finishes or drops out.

2. Enrollment levels and drop-out rates are the same for all years in the planning period, so for multi-year programs the annual cost for the program is the same as the cost of sending a cohort all the way through the program.

Constraint:

$$\sum_{j=1}^{J} \sum_{d=D,\bar{D}} \sum_{o=O,\bar{O}} \sum_{h=H,\bar{H}} C_j^{doh} \, x_j^{doh} \leq B$$

Remark: If program costs for the various combinations of "doh" cannot be differentiated, the constraint can be written as

$$\sum_{j=1}^{J} C_j \sum_{d=D,\bar{D}} \sum_{o=O,\bar{O}} \sum_{h=H,\bar{H}} \gamma_j^{doh} \leq B$$

where: $C_j$ = the average program cost per student in service area j for any personal category "doh."

## E. Total Number of Constraints

From sections:

A: J

B: $2^J$-1

C: 3

D: 1

Examples: the general case, and when J=7 and J=8

Total (J) = $2^J$ + J + 3

Total (J=7) = 128 + 7 + 3 = 138

Total (J=8) = 256 + 8 + 3 = 267

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